SAMPLE QUESTION PAPER (BASIC) - 07

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

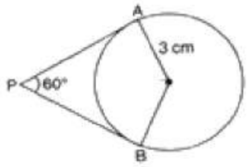
Section A

1. If the coordinates of a point are (3, -7), then its ordinate is **[1]**

a) 7 b) -3

c) -7 d) 3

2. If two tangents inclined at 60° are drawn to circle of radius 3 cm, then length of each tangent is equal to **[1]**



a) $3\sqrt{3}$ b) 3 cm

c) $2\sqrt{3}$ cm d) $3\sqrt{2}$ cm

3. From a well-shuffled deck of 52 cards, one card is drawn at random. What is the probability of getting a queen? **[1]**

a) None of these b) $\frac{4}{39}$

c) $\frac{1}{13}$ d) $\frac{1}{26}$

4. A circle drawn with origin as the centre passes through $(\frac{13}{2}, 0)$. The point which does not lie in the interior of the circle is **[1]**

a) $\frac{-3}{4}, 1$ b) $2, \frac{7}{3}$

c) $5, \frac{-1}{2}$ d) $\left(-6, \frac{5}{2}\right)$

5. The system of equations $6x + 3y = 6xy$ and $2x + 4y = 5xy$ has [1]

a) two solution
b) one unique solution
c) many solutions
d) no solution

6. If three points (0,0), $(3, \sqrt{3})$ and $(3, \lambda)$ form an equilateral triangle, then λ = [1]

a) -4
b) None of these
c) -3
d) 2

7. The probability of an impossible event is [1]

a) $\frac{1}{2}$
b) not defined
c) 0
d) 1

8. A sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder. If the radius of the sphere is r, then the volume of the cylinder is [1]

a) $2\pi r^3$
b) $8\pi r^3$
c) $\frac{8}{3}\pi r^3$
d) $4\pi r^3$

9. A die is thrown once. The probability of getting an even number is [1]

a) $\frac{1}{3}$
b) $\frac{5}{6}$
c) $\frac{1}{6}$
d) $\frac{1}{2}$

10. By a reduction of Re.1 per kg in the price of sugar, Radha can buy one kg sugar more for Rs.56. The original price of 1 kg of sugar is [1]

a) Rs.8
b) Rs.7
c) Rs.9
d) Rs.6

11. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is [1]

a) $\frac{1}{2}$
b) -2
c) $\frac{1}{4}$
d) 2

12. $\frac{1+\tan^2 A}{1+\cot^2 A} =$ [1]

a) 1
b) $\cot^2 A$
c) $\tan^2 A$
d) $\sec^2 A$

13. If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF, then the product of two numbers is [1]

a) 205400
b) 203400
c) 194400
d) 198400

14. Points (1, 0) and (-1, 0) lies on [1]

a) line $x + y = 0$
b) y-axis
c) x-axis
d) line $x - y = 0$

15. If the angle of depression of a car from a 100 m high tower is 45°, then the distance of the car from the tower is [1]

a) 100 m
b) 200 m
c) $100\sqrt{3}$ m
d) $200\sqrt{3}$ m

16. The marks obtained by 9 students in Mathematics are 59, 46, 31, 23, 27, 44, 52, 40 and 29. The mean of the data is **[1]**

a) 39
b) 38
c) 37
d) 40

17. HCF of $(2^3 \times 3^2 \times 5)$, $(2^2 \times 3^3 \times 5^2)$ and $(2^4 \times 3 \times 5^3 \times 7)$ is **[1]**

a) 60
b) 48
c) 30
d) 105

18. The sum of the numerator and denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to $\frac{1}{3}$. The fraction is **[1]**

a) $\frac{-7}{11}$
b) $\frac{5}{13}$
c) $\frac{-5}{13}$
d) $\frac{7}{11}$

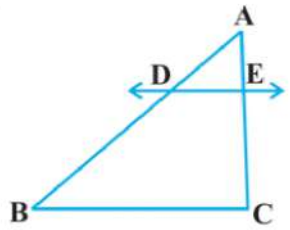
19. **Assertion (A):** H.C.F. of smallest prime and smallest composite is 2. **[1]**

Reason (R): Smallest prime is 2 and smallest composite is 4 so their H.C.F. is 2.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** D and E are points on the sides AB and AC respectively of a $\triangle ABC$ such that AD = 5.7 cm, DB = 9.5 cm, AE = 4.8 cm and EC = 8 cm then DE is not parallel to BC. **[1]**

Reason (R): If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side.



a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is:(i) red? (ii) not red? **[2]**

22. Half the perimeter of a rectangle garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden. **[2]**

OR

Solve the pair of linear equations by the substitution method: x + y = 14; x - y = 4.

23. Find a quadratic polynomial of the given numbers as the sum and product of its zeroes respectively. $-\frac{1}{4}, \frac{1}{4}$ **[2]**

24. Determine the ratio in which the line 2x + y - 4 = 0 divides the line segment joining the points A(2, -2) and B(3, **[2]**

7).

25. Find the length of the tangent drawn from a point whose distance from the centre of a circle is 25 cm. Given that radius of the circle is 7 cm. **[2]**

OR

Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

Section C

26. Prove that: sec A (1 - sin A) (sec A + tan A) = 1 **[3]**

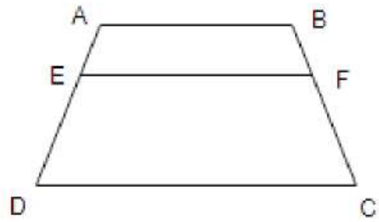
27. Solve the pair of linear equation: $\frac{x}{a} - \frac{y}{b} = 0$, ax + by = $a^2 + b^2$. **[3]**

28. Find the HCF and LCM of the following pairs of positive integers by applying the prime factorization method: 72, 90 **[3]**

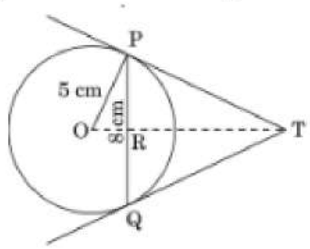
OR

On morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

29. ABCD is a trapezium with AB || DC. E and F are two points on non-parallel sides AD and BC respectively, such that EF is parallel to AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$ **[3]**



30. In a given figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP. **[3]**



OR

In an isosceles triangle ABC in which AB = AC = 6 cm is inscribed in a circle of radius 9 cm, find the area of the triangle.

31. If a 1.5-m-tall girl stands at a distance of 3m from a lamp-post and casts a shadow of length 4.5m on the ground then find the height of the lamp-post. **[3]**

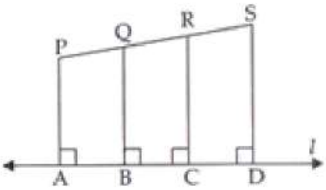
Section D

32. Solve for y: $4y^2 + 4qy - (p^2 - q^2) = 0$ **[5]**

OR

The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of field.

33. In the given figure, PA, QB, RC and SD are all perpendiculars to a line '*l*', AB = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm. Find PQ, QR and RS. **[5]**



34. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles is $\frac{24}{7}\text{cm}^2$. Find the radius of each circle. **[5]**

OR

A semicircular region and a square region have equal perimeters. The area of the square region exceeds that of the semicircular region by 4 cm^2. Find the perimeters and areas of the two regions.

35. The following table shows the ages of the patients admitted in a hospital during a year: **[5]**

Age (in years)	**5-15**	**15-25**	**25-35**	**35-45**	**45-55**	**55-65**
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

An ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day Sheetal and her brother came to his shop. Sheetal purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm. her brother purchased rectangular brick shaped ice cream with length 9 cm, width 4cm and thickness 2 cm.

(i) The volume of the ice-cream without hemispherical end.

(ii) The volume of the ice-cream with a hemispherical end.

(iii) Find the volume her brother ice cream?

OR

Whose quantity of ice cream is more and by how much?

37. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find an increase in the production of TV every year.

(ii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find in which year production of TV is 1000.

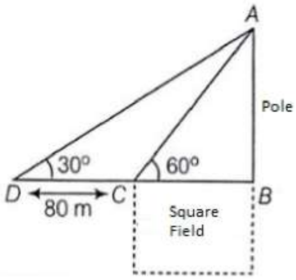
(iii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the 10th year.

OR

They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the total production in first 7 years.

38. **Read the text carefully and answer the questions:** **[4]**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is 60^{o}. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30^{o}.



(i) Find the height of the pole too so that he can arrange a ladder accordingly to put an effigy on the pole.

(ii) Find the length of his square field so that he can buy material to do the fencing work accordingly.

(iii) Find the Distance from Farmer at position C and top of the pole?

OR

Find the Distance from Farmer at position D and top of the pole?

Solution

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Class 10 - Mathematics

Section A

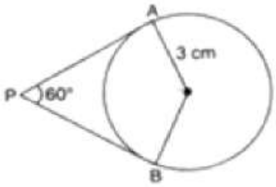
1. **(c)** -7

   Explanation: Since y-coordinate of a point is called ordinate. Its distance from the x-axis measured parallel to the y-axis Therefore, the ordinate is -7.

2. **(a)** $3\sqrt{3}$

   Explanation:

   Let O be the centre. Construction: Joined OP.

   Since OP bisects $\angle$P, therefore, $\angle\text{APO} = \angle\text{OPB} = 30^\circ$ And $\angle\text{OAP} = 90^\circ$

   $\therefore \tan 30^\circ = \frac{\text{OA}}{\text{AP}} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{\text{AP}}$

   $\Rightarrow \text{AP} = 3\sqrt{3}$ cm Since each tangent from an external point to a circle are equal.

   Therefore, PA = PB = $3\sqrt{3}$ cm

3. **(c)** $\frac{1}{13}$

   Explanation: Number of all possible outcomes = 52.

   Number of queens = 4.

   $\therefore$P (getting a queen) = $\frac{4}{52} = \frac{1}{13}$

4. **(d)** $\left(-6, \frac{5}{2}\right)$

   Explanation: Distance between (0, 0) and $\left(-6, \frac{5}{2}\right)$

   d = $\sqrt{(-6-0)^2 + \left(\frac{5}{2}-0\right)^2}$

   = $\sqrt{36 + \frac{25}{4}}$

   = $\sqrt{\frac{144+25}{4}}$

   = $\sqrt{\frac{169}{4}} = \frac{13}{2}$ = 6.5

   So, the point $\left(-6, \frac{5}{2}\right)$ does not lie in the circle.

5. **(b)** one unique solution

   Explanation: Given: 6x + 3y = 6xy

   $\Rightarrow \frac{3}{x} + \frac{6}{y}$ = 6

   And, 2x + 4y = 5xy

   $\Rightarrow \frac{4}{x} + \frac{2}{y}$ = 5

   Let $\frac{1}{x}$ = u and $\frac{1}{y}$ = v

   $\therefore$ The equations are 3u + 6v = 6 and 4u + 2v = 5

   Here, $a_1 = 3$, $a_2 = 4$, $b_1 = 6$, $b_2 = 2$, $c_1 = 6$ and $c_2 = 5$

   $\frac{a_1}{a_2} = \frac{3}{4}, \frac{b_1}{b_2} = \frac{6}{2} = \frac{3}{1}$ and $\frac{c_1}{c_2} = \frac{6}{5}$

   $\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

   Therefore, the system of given equations has a unique solution.

6. **(b)** None of these

   Explanation: Let the points (0, 0), $(3, \sqrt{3})$ and $(3, \lambda)$ from an equilateral triangle AB = BC = CA

   $\Rightarrow AB^2 = BC^2 = CA^2$

Now, $AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= (3-0)^2 + (\sqrt{3}-0)^2 = (3)^2 + (\sqrt{3})^2$
$= 9 + 3 = 12$
$BC^2 = (3-3)^2 + (\lambda - \sqrt{3})^2$
$= (0)^2 + (\lambda - \sqrt{3})^2 = (\lambda - 3)^2$
and $CA^2 = (0-3)^2 + (0-\lambda)^2 = (-3)^2 + (-\lambda)^2$
$= 9 + \lambda^2$
$AB^2 = CA^2 \Rightarrow 12 = 9 + \lambda^2$
$\Rightarrow \lambda^2 = 12 - 9 = 3$
$\therefore \lambda = \pm\sqrt{3}$

7. **(c)** 0

Explanation: An event which has no chance of occurrence is called an impossible event.

for example: The probability of getting more than 6 when a die is thrown is an impossible event because the highest number in a die is 6

The probability of an impossible event is always 0.

8. **(a)** $2\pi r^3$

Explanation: Volume of a sphere = $(4/3)\pi r^3$

Volume of a cylinder = $\pi r^2 h$

Given, sphere is placed inside a right circular cylinder so as to touch the top, base and lateral surface of the cylinder and the radius of the sphere is *r*.

Thus, height of the cylinder = diameter = 2r and base radius = r

Volume of the cylinder = $\pi \times r^2 \times 2r = 2\pi r^3$

9. **(d)** $\frac{1}{2}$

Explanation: Number of all possible outcomes = 6.

Even numbers are 2,4, 6. Their number is 3.

$\therefore$ P (getting an even number) = $\frac{3}{6} = \frac{1}{2}$

10. **(a)** Rs.8

Explanation: Let the original price of 1 kg sugar = Rs. x

$\therefore$ In Re. 1, the weight of sugar can be bought = $\frac{1}{x}$ kg

$\therefore$ In Rs. 56, the weight of sugar can be bought = $\frac{56}{x}$ kg

New price = Rs. (x - 1)

$\therefore$ In Rs. 56, the weight of sugar can be bought = $\frac{56}{x-1}$ kg

According to question, $\frac{56}{x-1} - \frac{56}{x} = 1$

$\Rightarrow \frac{56x-56x+56}{x(x-1)} = 1$

$\Rightarrow \frac{56}{x^2-x} = 1$

$\Rightarrow x^2 - x - 56 = 0$

$\Rightarrow x^2 - 8x + 7x - 56 = 0$

$\Rightarrow x(x - 8) + 7(x - 8) = 0$

$\Rightarrow (x + 7)(x - 8) = 0$

$\Rightarrow x + 7 = 0$ and $x - 8 = 0$

$\Rightarrow x = -7$ and $x = 8$ [x = -7 is not possible]

Therefore, the original price of 1 kg of sugar is Rs. 8

11. **(d)** 2

Explanation: If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$ then, substituting the value of $\frac{1}{2}$ in place of x should give us the value of k.

Given, $x^2 + kx - \frac{5}{4} = 0$ where, $x = \frac{1}{2}$

$\Rightarrow \left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$

$\Rightarrow \frac{k}{2} = \frac{5}{4} - \frac{1}{4}$

$\therefore k = 2$

12. **(c)** $\tan^2 A$

Explanation: $\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}$

$=\frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}}$

$=(1+\tan^2 A)\left(\frac{\tan^2 A}{\tan^2 A+1}\right)=\tan^2 A$

Hence, the correct choice is $\tan^2 A$.

13. **(c)** 194400

Explanation: Let the HCF of the numbers be x and their LCM be y.

It is given that the sum of the HCF and LCM is 1260, therefore

x + y = 1260 ....(i)

And, LCM is 900 more than HCF.

y = x + 900 ..... (ii)

Substituting (ii) in (i), we get:

x + x + 900 =1260

⇒ 2x + 900 = 1260

⇒ 2x = 1260 - 900

⇒2x = 360

⇒ x = 180

Substituting x = 180 in (i), we get:

y = 180 + 900

⇒ y = 1080

We also know that the product the two numbers is equal to the product of their LCM and HCF

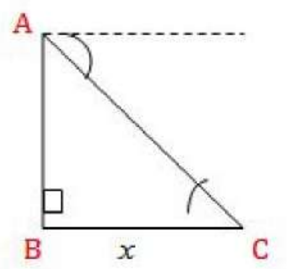
Thus, product of the numbers = 1080(180) = 194400

14. **(c)** x-axis

Explanation: Since the ordinates of given points are 0. Therefore, points lie on x-axis.

15. **(a)** 100 m

Explanation:



Let the distance of the car from

the tower be $x$meters.

$\therefore \tan 45^\circ=\frac{\mathrm{AC}}{\mathrm{BC}}$

$\Rightarrow 1=\frac{100}{x}$ m

$\Rightarrow x=100$ m

Therefore, the distance of the car from the tower is 100 m.

16. **(a)** 39

Explanation: Mean = $\frac{\text{Sum of all observations}}{\text{Number of observations}}$

$=\frac{59+46+31+23+27+44+52+40+29}{9}$

$=\frac{351}{9}$

= 39

17. **(a)** 60

Explanation: HCF = $(2^3\times3^2\times5, 2^2\times3^3\times5^2,\ 2^4\times3\times5^3\times7)$

HCF = Product of smallest power of each common prime factor in the numbers

$=2^2\times3\times5=60$

18. **(b)** $\frac{5}{13}$

Explanation: Let the fraction be $\frac{x}{y}$.

According to question

x + y = 18 ... (i)

And $\frac{x}{y+2} = \frac{1}{3}$

$\Rightarrow$ 3x = y + 2

$\Rightarrow$ 3x - y = 2 ... (ii)

On solving eq. (i) and eq. (ii), we get

x = 5, y = 13

Therefore, the fraction is $\frac{5}{13}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Smallest prime is 2 and smallest composite is 4 so H.C.F. of 2 and 4 is 4.

20. **(d)** A is false but R is true.

Explanation: If a line divides any two sides of a triangle in the same ratio then it is parallel to the third side. This is the Converse of the Basic Proportionality theorem.

So, the Reason is correct.

Now, $\frac{AD}{DB} = \frac{5.7}{9.5} = \frac{57}{95} = \frac{3}{5}$

and $\frac{AE}{EC} = \frac{4.8}{8} = \frac{48}{80} = \frac{3}{5}$

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

By Converse of Basic Proportionality theorem, DE||BC

So, the Assertion is not correct.

Section B

21. There are 3 + 5 = 8 balls in a bag. Out of these 8 balls, one can be chosen in 8 ways.

$\therefore$Total number of elementary events = 8

$Proabibilty\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$

i. Since the bag contains 3 red balls, therefore, one red ball can be drawn in 3 ways.

$\therefore$Favourable number of elementary events = 3

Hence P (getting a red ball) = $\frac{3}{8}$

ii. Since the bag contains 5 black balls along with 3 red balls, therefore one black (not red) ball can be drawn in 5 ways.

$\therefore$Favourable number of elementary events = 5

Hence P (getting "not a red ball") = $\frac{5}{8}$

22. Let length of rectangular garden = x metres

and width of rectangular garden = y metres

According to given conditions, perimeter = 36 m

$\frac{1}{2}$(2x + 2y) = 36

$\Rightarrow$ x + y = 36 ....(i)

and x = y+4

$\Rightarrow$ x - y =4 ....(ii)

Adding eq. (i) and (ii),

2x = 40 $\Rightarrow$ x = 20 m

Subtracting eq. (ii) from eq. (i),

2y = 32 $\Rightarrow$ y = 16 m

Hence, length = 20 m and width = 16 m

OR

x + y = 14 ....(1)

x - y = 4 ....(2)

x = 4 + y from equation (2)

Putting this in equation (1), we get

4+y +y =14

⇒ 2y =10

⇒ y = 5

Putting value of y in equation (1), we get

x + 5 = 14

⇒ x = 14 - 5 = 9

Therefore, x =9 and y =5

23. Let the polynomial be $ax^2 + bx + c$,

and its zeroes be α and β.

Then, $\alpha + \beta = -\frac{1}{4} = -\frac{b}{a}$ and $\alpha\beta = \frac{1}{4} = \frac{c}{a}$

If a = 4, then b = 1 and c = 1.

So, one quadratic polynomial which fits the given conditions is $4x^2 + x + 1$.

Aliter,

It given that $\alpha + \beta = -\frac{1}{4}$ and $\alpha\beta = \frac{1}{4}$

now, standard form of quadratic polynomial is given by $x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - \left(-\frac{1}{4}\right)x + \frac{1}{4}$

$= \frac{1}{4}\left(4x^2 + x + 1\right)$

Hence the required quadratic polynomial is $4x^2 + x + 1$

24. Let the ratio be K: 1

Coordinate of P are $\left(\frac{3K+2}{K+1}, \frac{7K-2}{K+1}\right)$

P lies on the line 2x + y - 4 = 0

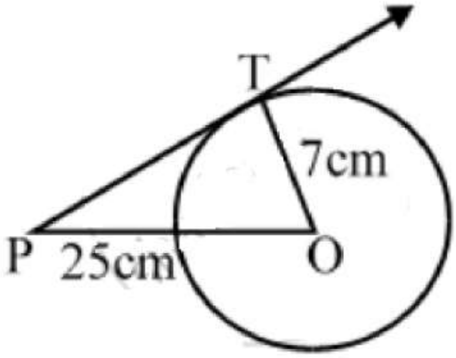
$\Rightarrow 2\left(\frac{3K+2}{K+1}\right) + \frac{7K-2}{K+1} - \frac{4}{1} = 0$

⇒ 6K + 4 + 7K - 2 - 4K - 4 = 0

⇒ 9K - 2 = 0

$\Rightarrow K = \frac{2}{9}$ or 2 : 9

25.



Let O is the centre of the circle and P is a point such that OP = 25 cm and PT is the tangent to the circle.

$OT = \text{radius} = 7cm$

In △OTP, we have ∠T = 90°

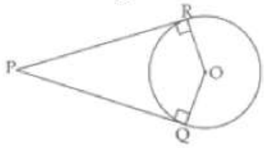
$\mathrm{OP}^2 = \mathrm{OT}^2 + \mathrm{PT}^2$ [By using Pythagorean theorem]

$\Rightarrow (25)^2 = 7^2 + \mathrm{PT}^2 \Rightarrow \mathrm{PT}^2 = 625 - 49 = 576$

$\Rightarrow PT = 24cm$

OR

Given: Tangents PR and PQ from an external point P to a circle with centre O.



To prove: Quadrilateral QORP is cyclic.

Proof: RO and RP are the radius and tangent respectively at contact point R.

Therefore, ∠PRO = 90°

Similarly ∠PQO = 90°

In quadrilateral QOPR, we have

∠P + ∠R + ∠O + ∠Q = 360°

⇒ ∠P + ∠90° + ∠O + ∠90° = 360°

$\Rightarrow \angle P + \angle O = 360° - 180° = 180°$

These are opposite angles of quadrilateral QORP and are supplementary.

Therefore, Quadrilateral QORP is cyclic. hence, proved.

Section C

26. L.H.S.

= sec A(1 - sin A)(sec A + tan A)

$= \frac{1}{\cos A}(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$

$= \frac{(1-\sin A)}{\cos A}\left(\frac{1+\sin A}{\cos A}\right)$

$= \frac{(1-\sin A)(1+\sin A)}{\cos A \times \cos A}$

$= \frac{(1^2-\sin^2 A)}{\cos^2 A}$.[Since, $(a - b)(a + b) = a^2 - b^2$]

$= \frac{(1-\sin^2 A)}{\cos^2 A}$

$= \frac{\cos^2 A}{\cos^2 A}$

= 1

=RHS

Hence, proved.

27. The given pair of linear equation is

$\frac{x}{a} - \frac{y}{b} = 0$ ....(1)

$ax + by = a^2 + b^2$ ....(2)

From equation (1),

$\frac{y}{b} = \frac{x}{a}$

$\Rightarrow y = \frac{b}{a}x$ ....(3)

Substituting the value of y in equation (2), we get

$ax + b\left(\frac{b}{a}x\right) = a^2 + b^2$

$\Rightarrow \frac{a^2+b^2}{a}x = a^2 + b^2$

$\Rightarrow x = \frac{a(a^2+b^2)}{a^2+b^2} = a$

Substituting this value of x in equation (3), we get $y = \frac{b}{a}(a) = b$

Hence, the solution of the given pair of linear equations is

x = a, y = b.

Verification : Substitution x = a, y = b,

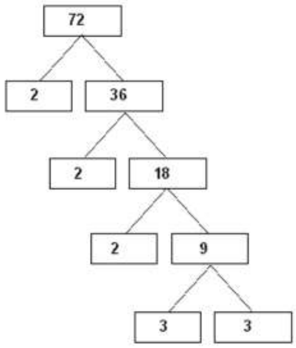
We find that both the equations (1) and (2) are satisfied as shown below:

$\frac{x}{a} - \frac{y}{b} = \frac{a}{a} - \frac{b}{b} = 1 - 1 = 0$

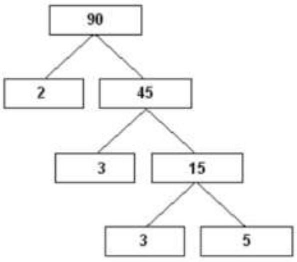
$ax + by = a(a) + b(b) = a^2 + b^2$

This verifies the solution.

28.



So, $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$



So, $90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

Therefore,

HCF (72, 90) = $2 \times 3^2 = 18$

LCM (72, 90) = $2^3 \times 3^2 \times 5 = 360$

OR

Since, the three persons start walking together.

$\therefore$ The minimum distance covered by each of them in complete steps = LCM of the measures of their steps

$40 = 8 \times 5 = 2^3 \times 5$

$42 = 6 \times 7 = 2 \times 3 \times 7$

$45 = 9 \times 5 = 3^2 \times 5$

Hence LCM (40, 42, 45)

$= 2^3 \times 3^3 \times 5 \times 7 = 8 \times 9 \times 5 \times 7 = 2520$

$\therefore$ The minimum distance each should walk so that each can cover the same distance

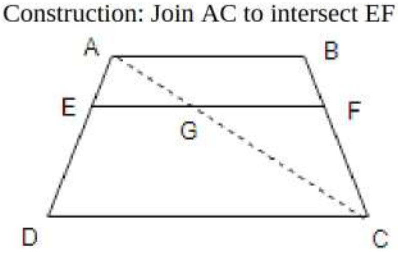
= 2520 cm = 25.20 meters.

29. Given, In trapezium ABCD,

$AB||DC$ and $EF||DC$

To prove $\frac{AE}{ED} = \frac{BF}{FC}$

Construction: Join AC to intersect EF at G.



Proof Since, $AB||DC$ and $EF||DC$

$EF||AB$ [since, lines parallel to the same line are also parallel to each other]...... (i)

In $\triangle ADC$, $EG||DC$ [$\because EF||DC$]

By using basic proportionality theorem,

$\frac{AE}{ED} = \frac{AG}{GC}$ ....(ii)

In $\triangle ABC$, $GF||AB$ [$\because EF||AB$ from (i)]

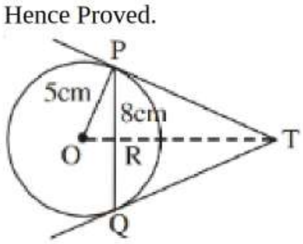
By using basic proportionality theorem ,

$\frac{CG}{AG} = \frac{CF}{BF}$ or $\frac{AG}{GC} = \frac{BF}{CF}$ [On taking reciprocal of the terms]............. (iii)

From Equations (ii) and (iii), we get

$\frac{AE}{ED} = \frac{BF}{FC}$

Hence Proved.

30. 

Let TR be x cm and TP be y cm

OT is perpendicular bisector of PQ

So PR = 4 cm (PR = $\frac{PQ}{2} = \frac{8}{2}$)

In $\triangle$OPR, $OP^2 = PR^2 + OR^2$

$5^2 = 4^2 + OR^2$

$OR = \sqrt{25 - 16}$

$\therefore OR = 3$ cm

In $\triangle PRT$, $PR^2 + RT^2 = PT^2$

$y^2 = x^2 + 4^2$ .....(1)

In $\triangle OPT$, $OP^2 + PT^2 = OT^2$

$(x + 3)^2 = 5^2 + y^2$ (OT = OR + RT = 3 + x)

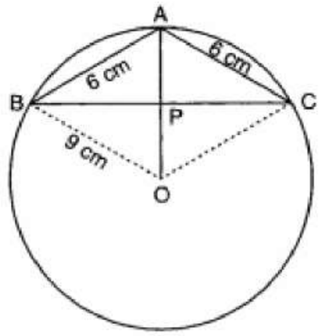
$\therefore (x + 3)^2 = 5^2 + x^2 + 16$ [using (1)]

Solving, we get $x = \frac{16}{3}$ cm

From (1), $y^2 = \frac{256}{9} + 16 = \frac{400}{9}$

So, $y = \frac{20}{3}$cm = 6.667 cm

OR



Let O be the centre of the circle and let P be the mid-point of BC. Then ,$OP \perp BC$

Since Δ ABC is isosceles and P is the mid-point of BC. Therefore, $AP \perp BC$ as median from the vertex in an isosceles triangle is perpendicular to the base.

Let A P = x and PB = CP = y.

Applying Pythagoras theorem in Δ APB and Δ OPB, we have

$AB^2 = BP^2 + AP^2$

and, $OB^2 = OP^2 + BP^2$

$\Rightarrow 36 = y^2 + x^2$ ....(i)

and, $81 = (9 - x)^2 + y^2$ ...(ii)

$\Rightarrow 81 - 36 = [(9 - x)^2 + y^2] - (y^2 + x^2)$[Subtracting (i) from (ii)]

$\Rightarrow 45 = 81 - 18x$

$\Rightarrow x = 2$ cm, therefore AP = x = 2 cm .....(iii)

Putting x = 2 in (i), we get

$36 = y^2 + 4 \Rightarrow y^2 = 32 \Rightarrow y = 4\sqrt{2}$ cm

Therefore,BC = 2BP = 2y = $8\sqrt{2}$ .cm .....(iv)

Therefore,

Area of Δ ABC

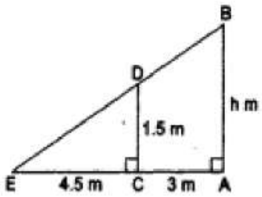
$= \frac{1}{2}BC \times AP$

$= \frac{1}{2} \times 2y \times x$ [from (iii) & (iv)]

= xy

$= 2 \times 4\sqrt{2} = 8\sqrt{2}$ cm^2.

31. Let AB be the lamp-post and CD be the girl.



Let CE be the shadow of CD. Then,

$CD = 1.5m, CE = 4.5m \; and \; AC = 3m.$

Let AB = h m.

Now, ΔAEB and ΔCED are similar.

$\therefore \quad \frac{AB}{AE} = \frac{CD}{CE} \Rightarrow \frac{h}{(3+4.5)} = \frac{1.5}{4.5} = \frac{1}{3}$

$\Rightarrow h = \frac{1}{3} \times 7.5 = 2.5$

Section D

32. In equation, $4y^2 + 4qy - (p^2 - q^2) = 0$

A = 4, B = 4q and C = $q^2 - p^2$

$y = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$= \frac{-4q \pm \sqrt{(4q)^2 - 4 \times 4(q^2 - p^2)}}{2 \times 4}$

$= \frac{-4q \pm \sqrt{16q^2 - 16q^2 + 16p^2}}{8}$

$= \frac{-4q \pm 4p}{8}$

$= -\frac{(p+q)}{2}, \frac{(p-q)}{2}$

$\therefore$ the roots = $-\frac{(p+q)}{2}, \frac{(p-q)}{2}$

OR

Let the shorter side of the rectangular field be x metres.

Then, the longer side of the rectangular field = (x + 30) metres

Therefore, the diagonal of the rectangular field =

$= \sqrt{(Length\ of\ the\ shorter\ side)^2 + (Length\ of\ the\ longer\ side)^2}$ [By Pythagoras theorem]

$= \sqrt{x^2 + (x+30)^2}$ metres

According to the question, $\sqrt{x^2 + (x+30)^2} = x + 60$

Squaring both sides,we get

$x^2 + (x+30)^2 = (x+60)^2$

$\Rightarrow x^2 + x^2 + 60x + 900$

$= x^2 + 120x + 3600$

$\Rightarrow x^2 - 60x - 2700 = 0$

which is a quadratic equation in x.

Here, a = 1, b = -60, c = -2700

Using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

we get $= \frac{-(-60) \pm \sqrt{(-60)^2 - 4(1)(-2700)}}{2(1)}$

$= \frac{60 \pm \sqrt{3600 + 10800}}{2} = \frac{60 \pm \sqrt{14400}}{2}$

$= \frac{60 \pm 120}{2} = \frac{60+120}{2}, \frac{60-120}{2}$

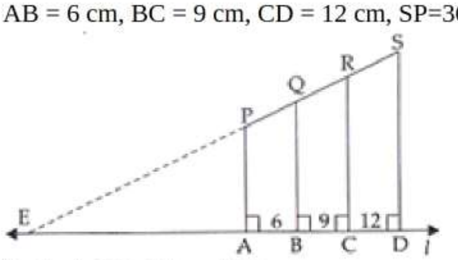
$\Rightarrow x = 90, -30$

Since, x cannot be negative, being a dimension, the length of the shorter side of the rectangular field is 90 metres.

The length of the longer side = x + 30 = 90 + 30 = 120 metres

33. **Given:** PA, QB, RC and SD are perpendicular on line l.

AB = 6 cm, BC = 9 cm, CD = 12 cm, SP=36 cm



To find: PQ, QR and RS.

Construction: we produce SP so that it joins l at E.

Proof: In ΔEDS,

AP || BQ || CR || SD [Given]

∴ PQ : QR : RS = AB : BC : CD

PQ : QR : RS = 6 : 9 : 12

Let PQ = 6x

then QR = 9x

and RS = 12x

Now, PQ + QR + RS = 36 cm (given)

⇒ 6x + 9x + 12x = 36

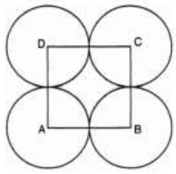
⇒ 27x = 36

$\Rightarrow x = \frac{36}{27} = \frac{4}{3}$

Therefore, PQ = 6 × $\frac{4}{3}$ = 8 cm

QR = 9 × $\frac{4}{3}$ = 12 cm

RS = 12 × $\frac{4}{3}$ = 16 cm

34. 

Let r cm be the radius of each circle.

Area of square - Area of 4 sectors = $\frac{24}{7}\text{cm}^2$

$(\text{side})^2$ - 4 $\left[\frac{\theta}{360}\pi r^2\right] = \frac{24}{7}\text{cm}^2$

or, $(2r)^2 - 4\left(\frac{90^\circ}{360^\circ} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - 4\left(\frac{1}{4^\circ} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - \left(\pi r^2\right) = \frac{24}{7}$

or, $4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$

or, $\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$

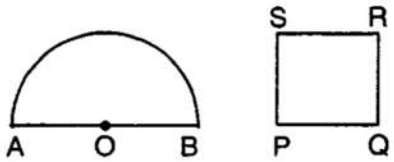
or, $6r^2 = 24$

or, $r^2 = 4$

or, $r = \pm 2$

or, Radius of each circle is 2 cm (r cannot be negative)

OR



Let radius of semicircular region be r units.

Perimeter = 2r + πr

Let side of square be x units

Perimeter = 4x units.

A.T.Q, 4x = 2r + πr $\Rightarrow x = \frac{2r+\pi r}{4}$

Area of semicircle = $\frac{1}{2}\pi r^2$

Area of square = x^2

A.T.Q, $x^2 = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow \left(\frac{2r+\pi r}{4}\right)^2 = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow \frac{1}{16}\left(4r^2 + \pi^2 r^2 + 4\pi r^2\right) = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow 4r^2 + \pi^2 r^2 + 4\pi r^2 = 8\pi r^2 + 64$

$\Rightarrow 4r^2 + \pi^2 r^2 - 4\pi r^2 = 64$

$\Rightarrow r^2\left(4 + \pi^2 - 4\pi\right) = 64$

$\Rightarrow r^2(\pi - 2)^2 = 64$

$\Rightarrow r = \sqrt{\frac{64}{(\pi-2)^2}}$

$\Rightarrow r = \frac{8}{\pi - 2} = \frac{8}{\frac{22}{7} - 2}$ = 7 cm

Perimeter of semicircle =$2 \times 7 + \frac{22}{7} \times 7$= 36 cm

Perimeter of square = 36 cm

Side of square = $\frac{36}{4}$ = 9 cm

Area of square = 9$\times$ 9 = 81 cm^2

Area of semicircle = $\frac{\pi r^2}{2} = \frac{22}{2\times 7} \times 7 \times 7$ = 77 cm^2

35. Mode:

Here, the maximum frequency is 23 and the class corresponding to this frequency is 35 - 45.

So, the modal class is 35 - 45.

Now, size (h) = 10

lower limit it (l) of modal class = 35

frequency (f_1) of the modal class = 23

frequency (f_0) of class previous the modal class = 21

frequency (f_2) of class succeeding the modal class = 14

$\therefore$ Mode = l + $\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times$ h = 35 + $\frac{23-21}{2\times 23 - 21 - 14} \times$ 10

= 35 + $\frac{2}{11} \times$ 10 = 35 + $\frac{20}{11}$

= 35 + 1.8 (approx.)

= 36.8 years (approx.)

Mean:-

Take a = 40, h = 10.

Age (in years)	**Number of patients (f_i)**	**Class marks (x_i)**	**$d_i = x_i - 40$**	$u_i = \frac{x_i - 40}{10}$	**$f_i u_i$**
5-15	6	10	–30	–3	–18
15-25	11	20	–20	–2	–22
25-35	21	30	–10	–1	–21
35-45	23	40	0	0	0
45-55	14	50	10	1	14
55-65	5	60	20	2	10
Total	$\sum f_i = 80$				$\sum f_i u_i$ = - 37

Using the step deviation method,

$\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times$ h = 40 + $\left(\frac{-37}{80}\right) \times$ 10

= 40 - $\frac{37}{8}$ = 40 - 4.63

= 35.37 years

Interpretation:- Maximum number of patients admitted in the hospital are of the age 36.8 years (approx.), while on an average the age of a patient admitted to the hospital is 35.37 years.

Section E

36. **Read the text carefully and answer the questions:**

An ice-cream seller used to sell different kinds and different shapes of ice-cream like rectangular shaped with one end hemispherical, cone-shaped and rectangular brick, etc. One day Sheetal and her brother came to his shop. Sheetal purchased an ice-cream which has the following shape: ice-cream cone as the union of a right circular cone and a hemisphere that has the same (circular) base as the cone. The height of the cone is 9 cm and the radius of its base is 2.5 cm. her brother purchased rectangular brick shaped ice cream with length 9 cm, width 4cm and thickness 2 cm.

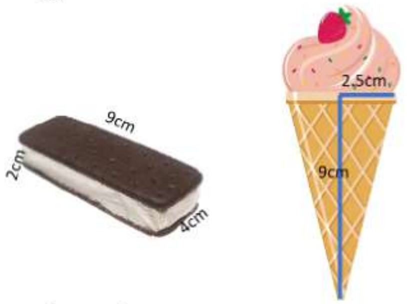
(i) For cone, radius of the base (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14} \text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

The volume of the ice-cream without hemispherical end = Volume of the cone

$= \frac{825}{14}\text{cm}^3$

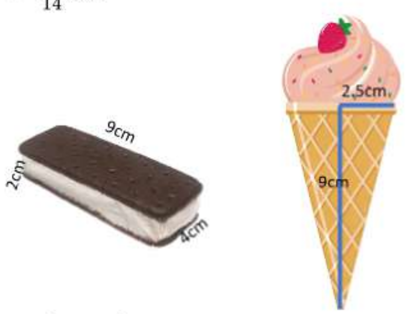
(ii) For cone, radius of the base (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere

$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$

$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$

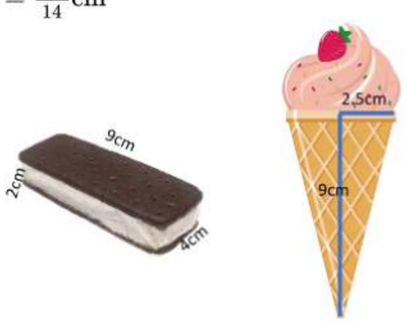
(iii)For cone, Radius of the base (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42} \text{cm}^3$

Volume of rectangular brick shaped ice cream = $9 \times 4 \times 2 = 72 \text{ cm}^3$

OR

For cone, Radius of the base (r)

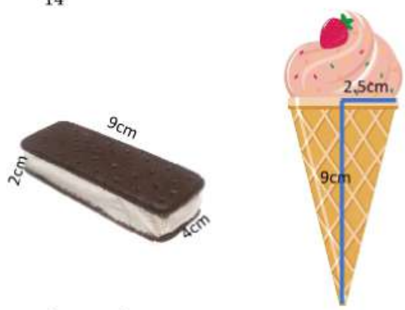
$= 2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

Sheetal ice cream quantity is more than her brother

Volume of Sheeta's ice cream - Volume her brother's ice cream

$= 91.66 - 72 = 19.66 \text{ cm}^3$

37. **Read the text carefully and answer the questions:**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We have, $a_3 = 600$ and

$a_3 = 600$

$\Rightarrow 600 = a + 2d$

$\Rightarrow a = 600 - 2d$ ...(i)

$\Rightarrow a_7 = 700$

$\Rightarrow a_7 = 700$

$\Rightarrow 700 = a + 6d$

$\Rightarrow a = 700 - 6d$ ...(ii)

From (i) and (ii)

$600 - 2d = 700 - 6d$

$\Rightarrow 4d = 100$

$\Rightarrow d = 25$

(ii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We know that first term = a = 550 and common difference = d = 25

$a_n = 1000$

$\Rightarrow 1000 = a + (n - 1)d$

$\Rightarrow 1000 = 550 + 25n - 25$

$\Rightarrow 1000 - 550 + 25 = 25n$

$\Rightarrow 475 = 25n$

$\Rightarrow n = \frac{475}{25} = 19$

(iii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year. The production in the 10th term is given by a_{10}. Therefore, production in the 10th year = $a_{10} = a + 9d = 550 + 9 \times 25 = 775$. So, production in 10th year is of 775 TV sets.

OR

Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year. Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$

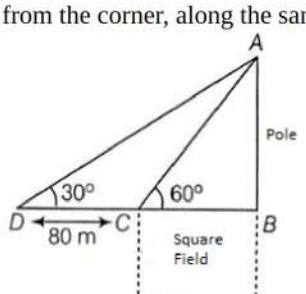
$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$

$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$

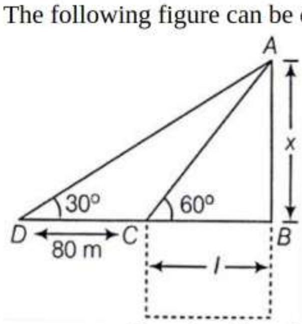
$\Rightarrow S_7 = 4375$

38. **Read the text carefully and answer the questions:**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is 60^o. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30^o.



(i) The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

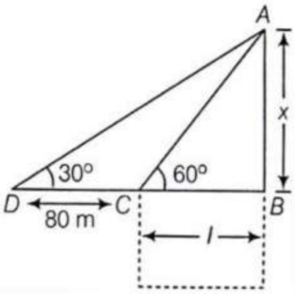
Now, l = 40 metres

We get,

$x = \sqrt{3}\, l = 40\sqrt{3} = 69.28$

Thus, height of the pole is 69.28 metres.

(ii) The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

In △ABC,

$\tan 60^o = \frac{x}{l}$

$\sqrt{3} = \frac{x}{l}$

x = $\sqrt{3}$ l ...(i)

Now, in △ABD,

$\tan 30^o = \frac{x}{80+l}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}l}{80+l}$ (From eq(i))

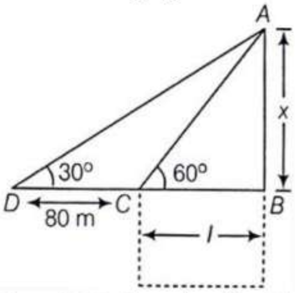
80 + l = 3l

2l = 80

l = 40

Thus, length of the field is 40 metres.

(iii)The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

Distance from Farmer at position C and top of the pole is AC.

In △ABC

$\cos 60^o = \frac{CB}{AC}$

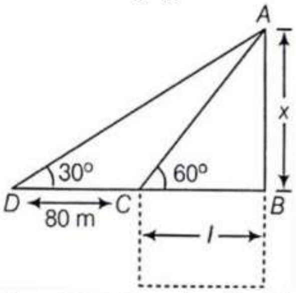
$\Rightarrow AC = \frac{CB}{\cos 60^0}$

$\Rightarrow AC = \frac{40}{\frac{1}{2}}$

⇒ AC = 80 m

OR

The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

Distance from Farmer at position D and top of the pole is AD

In △ABC

$\cos 30^o = \frac{DB}{AD}$

$\Rightarrow AD = \frac{DB}{\cos 30^0}$

$\Rightarrow AD = \frac{120}{\frac{\sqrt{2}}{2}} = \frac{240}{\sqrt{3}}$

$\Rightarrow$ AC = 138.56 m